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STEADY STATE HEAT TRANSFER WITHIN POROUS MEDIUM WITH TEMPERATURE DEPENDENT HEAT GENERATION

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Abstract—A theoretical analysis of internally energised porous reactor is presented. Constant and temperature-dependent rate of heat generation are assumed. The fluid passed through the heated porous element medium changes phase from liquid to vapour and the vapour is further superheated. It is assumed that the regions of different phases are "separated" by two phase change "interfaces", the first of which denotes the average distance where evaporation starts and the second denotes the average distance where complete evaporation is reached.

The characteristics of the various parameters affecting the performance of a hollow cylindrical porous element are evaluated and presented for a representative range of the viscous flow regime where the inertia forces are neglected.

The concept of generating temperature-dependent rate of heat within the porous element is aimed at gaining stability in operation and protecting the solid from burnout.

NOMENCLATURE

C_1, C_2, constants of integration;
C_p, specific heat;
$C_{p_{l,v}}$, liquid to vapor specific heat ratio (C_{p_l}/C_{p_v});
D, dimensionless group $(= C_p\dot{m}/2\pi k_T)$;
F, temperature factor $[= \beta(T_0 - T_i)]$, equations (4, 7);
J_a, Jakob number $[= \lambda\rho_v/C_{p_l}\rho_l(T_0 - T_i)]$;
k, thermal conductivity;
k_T, effective conductivity of the saturated matrix;
$\dot{m}$, mass flow rate;
$\dot{M}$, dimensionless mass flow rate $[= \dot{m}/\rho\bar{v}(r_0 \cdot r_i)]$;
M, dimensionless group, equations (15b);
N, dimensionless rate of heat generation $\left[= \dfrac{q_r(r_0 - r_i)^2}{k_T(T_0 - T_i)}\right]$;
p, pressure;
P, dimensionless pressure $[= (p - p_0)/(p_i - p_0)]$;
Pe, Peclet number $[= \bar{v}(r_0 - r_i)/\alpha_l]$;
q, rate of heat generation per unit time per unit volume;
q_r, rate of heat generation at the reference temperature, T_r;
r, radial coordinate;
R, dimensionless radial coordinate $[= r/(r_0 - r_i)]$;
T, temperature;
T_r, reference temperature $(= T^*)$;
v_r, radial velocity;
$\bar{v}$, overall average Darcy-velocity $\left(= \dfrac{\kappa}{\mu_l}\dfrac{p_i - p_0}{r_i - r_0}\right)$, equation (4);
V, dimensionless Darcy-velocity $(= v_r/\bar{v})$;
$\bar{V}$, voltage, equation (2a).

Greek symbols

α, thermal diffusivity;
β, temperature-coefficient of electrical resistance;
ρ, density;
$\rho_{l,v}$, liquid–vapor density ratio $(= \rho_l/\rho_v)$;
κ, permeability of porous structure;
λ, latent heat of vaporization;
Ω, electrical resistance;
θ, dimensionless temperature $[= (T - T^*)/(T_0 - T_i)]$;
μ, viscosity;
ν, kinematic viscosity $(= \mu/\rho)$;
$\nu_{l,v}$, liquid–vapor kinematic viscosity ratio $(= \nu_l/\nu_v)$.

Subscripts and superscripts

av, average value;
c, start of evaporation;
d, end of evaporation;
f, fluid (liquid or vapor);
i, internal side of cylinder;
l, liquid;
o, external side of cylinder;
r, radial, or at the reference temperature;
T, effective property;
v, vapor;
$*$, saturation state;
j, number of iteration.

INTRODUCTION

THE INTEREST in the study of convective two-phase heat transfer in porous media arises from its wide spectrum of engineering applications. Most of the previous studies are related to drying or transpiration cooling systems [1–10], which are usually subjected

to a prescribed temperature or heat flux at their surfaces. These solutions do not apply to problems involving distributed heat sources within the porous medium. There appears to be little information available on the important problem of coupled heat and mass transfer in porous media with internal heat generation [11–15].

The concept of internally energised porous medium is of interest in various application areas. These include nuclear fuel, cladding and shielding, solar collectors, compact regenerator design, boiling water reactor design etc. Also, the short residence time for fluid passing through the porous element is a feature which may lead to its use in various chemical engineering processes.

The present study deals with the use of fluid flow through internally heated porous medium, in particular in connection with the novel methods of energy production. The solid particles forming the porous structure may be nuclear or electrically heated. Heat may be generated also due to the absorption of radiation. The fluid passed through the heated porous medium, may change phase from liquid to vapor and the vapor be further superheated. The enormous specific surface area enables high specific ratings even with small temperature driving forces between the solid and the fluid. The principle may prove useful where high coolant rates are required, or be applied for the production of steam of variable controlled quality (e.g. in food processing and pharmaceuticals).

Uniform distribution of fluid flow into the porous structure is essential for a stable operation. In order to achieve a uniform flow distribution, it was proposed in previous studies to feed the fluid stream through a dispenser (a porous body of a smaller permeability) before being fed to the heated porous body. However, the pore structure of a permeable material may be variable on a "microscopic" scale, and local non-uniformity of flow almost always may be resulted. The material area being starved of coolant would rise in temperature and the consequent excursion of temperature leads to the burn out of the element. Hence, new methods are required to ensure the stable operation of an internally heated porous element.

The method presented in this study is based on generating temperature-dependent rate of heat characterized by a decreasing function with the temperature. Thus, as the temperature rises, the rate of heat generation decreases and the temperature falls-down. The concept of generating internal heat of a temperature-dependent rate seems to be promising in concern to stability of operation and long-life element due to the dynamic self-control, which such an element possesses.

It is reasonable to conceive of two basic geometries for such a porous element: The plain and the cylindrical where in the later the flow may be from either the inside or the outside. In recent years, sintered porous elements have become available in the form of hollow cylinders (or tubes). These have greatly increased their range of application. This brings the author to analyse the cylindrical shape.

THE THEORETICAL MODEL AND GOVERNING EQUATIONS

Consider a unit of a hollow porous cylinder, Fig. 1. The inner and outer radii are r_i and r_o, respectively. Liquid at temperature T_i and pressure P_i is continuously fed to the center of the cylinder. The liquid is assumed to flow radially outwards through the porous medium

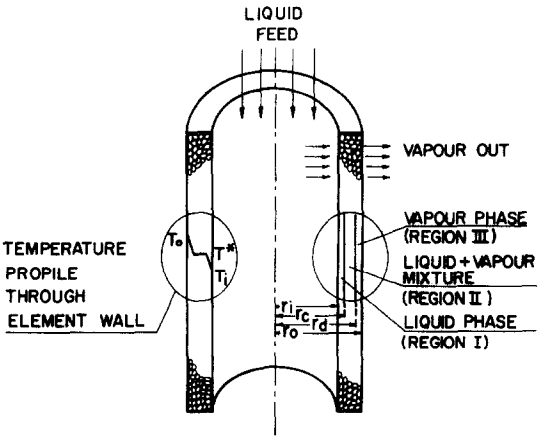

FIG. 1. Schematic presentation of physical system and coordinates.

by an imposed total pressure gradient $(p_i - p_o)$, where p_o is a constant pressure maintained at the outer surface of the cylinder. In the pressure gradients considered here, the flow is slow enough so that the viscous forces dominate over the inertia forces. Because of the complex structure of the porous medium it is impossible to formulate the problem in terms of the actual flow through the pores. As in most studies on the viscous regime flow through porous medium, the Darcy's law is applicable:

$$v_r = -\frac{\kappa}{\mu_f}\frac{\mathrm{d}p}{\mathrm{d}r} \tag{1}$$

where κ is the permeability of the porous material, μ_f is the fluid viscosity, v_r and $\mathrm{d}p/\mathrm{d}r$ are the radial velocity and the radial pressure gradient. The liquid passed through the heated porous medium, is firstly heated to saturation state (region I), changes phase from liquid to vapor (region II), and the vapor is further superheated (region III) to an exit temperature T_0, greater than the saturation temperature of the liquid corresponding to the pressure p_o. The temperature and pressure at the evaporation region are the saturation temperature and pressure T^* and p^* respectively, where $p_o < p^* < p_i$ and $T^*(p_i) < T^*(p^*) < T^*(p_o)$. For the case under consideration where equation (1) is applicable, the total pressure gradient is relatively small. Hence, $T^*(p_i) \approx T^*(p_o)$, and the saturation temperature at p^* is firstly approximated by $T^*((p_i + p_o)/2)$, and is corrected by solving for the pressure distribution (see below).

Consistent with slow flow through the porous medium it is reasonable to assume that the temperature of the solid and the adjacent fluid are equal. Thus the heterogeneous solid–fluid system is treated as a con-

tinuum, which allows average or "macroscopic" governing equations to be applied. In order to apply an average energy equation, it is necessary to determine an effective thermal conductivity of the saturated porous medium. Hashin and Shtrikman [17] derived an equation for the upper and lower bounds of the effective conductivity of heterogeneous materials, and [18–24] discussed the prediction of effective conductivities. For the problem under consideration, the upper bound from [17] is utilized to evaluate the effective conductivity for the saturated matrix. (Taking an upper and lower bound will yield limits on the solution of the problem.) The average energy equation can be written in the form [7, 25]:

$$\rho_f C_{pf} v \frac{\partial T}{\partial r} = \frac{\partial}{\partial r}\left(k_T \frac{\partial T}{\partial r}\right) + q \tag{2}$$

where k_T is the effective conductivity (consisting with assuming same fluid and solid temperatures) and q is the rate of heat generated within the porous structure per unit time per unit volume. In the case of electrical heating, q is usually given by:

$$q = \frac{\bar{V}^2/\Omega_r}{1+\beta(T-T_r)} = \frac{q_r}{1+\beta(T-T_r)} \tag{2a}$$

where $\bar{V}$ is the electrical voltage, T_r is a reference temperature, Ω_r is the electrical resistance at T_r, and β is the temperature-coefficient for Ω_r. Hence, q_r is the rate that would have been generated at temperature T_r.

The equation of continuity may be written in the form [7, 26]:

$$\frac{\partial(\rho_f r v_r)}{\partial r} = 0. \tag{3}$$

With reference to the following dimensionless variables:

$$R = r/(r_o - r_i) \quad P = (p-p_o)/(p_i-p_o) \quad \theta = (T-T^*)/(T_o-T_i)$$

$$\bar{v} = -\frac{\kappa}{\mu_l}\frac{p_i-p_o}{r_i-r_o} \quad V = v/\bar{v} \quad \dot{M} = \dot{m}/[\rho\bar{v}(r_o-r_i)]$$

$$J_a = \frac{\lambda\rho_v}{C_{p_l}\rho_l(T_0-T_i)} \quad Pe = \frac{\bar{v}(r_o-r_i)}{\alpha_l} \tag{4}$$

$$D = \frac{C_p\dot{m}}{2\pi k_T} \quad N = \frac{q_r(r_o-r_i)^2}{(T_0-T_i)k_T} \quad F = \beta(T_0-T_i)$$

$$\rho_{l,v} = \rho_l/\rho_v \quad \nu_{l,v} = \nu_l/\nu_v \quad C_{p_{l,v}} = C_{p_l}/C_{p_v}$$

the continuity, motion and energy equations are transformed to the dimensionless forms:

$$\frac{\partial}{\partial R}(VR) = 0 \tag{5}$$

$$V = -\frac{dP}{dR} \tag{6}$$

$$\frac{d^2\theta}{dR^2} + \frac{1-D}{R}\frac{d\theta}{dR} + \frac{N}{1+F\theta} = 0. \tag{7}$$

The boundary conditions applicable to this problem are:

$$R = R_i \qquad P = 1.0 \qquad \theta = \theta_i \tag{8a}$$

$$R = R_c \text{ (or } R_d) \qquad P = P^* \qquad \theta = 0 \tag{8b}$$

$$R = R_o \qquad P = 0 \qquad \theta = \theta_o \tag{8c}$$

where, R_c and R_d are the "interfaces" position. See Fig. 1. Actually, because of the thermal conductivity of the fluid, the fluid temperature will probably rise slightly before it enters the element at $R = R_i$. However this effect was found negligible [27]. Thus, the temperature at $R = R_i$ is assumed here to remain constant at the inlet value, θ_i. (Note that, the dispenser which is usually placed before the heated porous element provides thermal insulation between the feed and the heated element.) The conditions at the outer radius, R_o are maintained by the external vapor chamber. The evaporation in the intermediate region is assumed to proceed at constant temperature and pressure. This is reasonably valid in the case under consideration (in as much the total pressure gradient is by itself relatively small).

The simultaneous solution of motion, energy and continuity equations [equations (5)–(7)] yields the temperature distribution within the medium, the mass flux (or the heat load of the porous reactor) and the position of the phase-change "interfaces" (i.e. r_c and r_d).

Energy balances

The physical variables of the system may be furthermore inter-related by applying energy balances over the three regions (see Fig. 1). For the sake of clarity these are brought in the dimensional forms:

$$\dot{m}C_{p_l}(T^*-T_i) = 2\pi\int_{r_i}^{r_c} qr\,dr - 2\pi k_{T_l} r\left.\frac{dT}{dr}\right|_{r=r_i} \quad \text{(Region I)} \tag{9a}$$

$$\dot{m}\lambda = 2\pi\int_{r_c}^{r_d} qr\,dr + 2\pi k_{T_v} r\left.\frac{dT}{dr}\right|_{r=r_d} \quad \text{(Region II)} \tag{9b}$$

$$\dot{m}[C_{p_l}(T^*-T_i) + \lambda + C_{p_v}(T_0-T^*)] = 2\pi\int_{r_i}^{r_0} qr\,dr - 2\pi k_{T_l} r\left.\frac{dT}{dr}\right|_{r=r_i} \quad \text{(Overall balance)} \tag{9c}$$

$$\dot{m}C_{p_v}(T_0-T^*) = 2\pi\int_{r_d}^{r_0} qr\,dr - 2\pi k_{T_v} r\left.\frac{dT}{dr}\right|_{r=r_d} \quad \text{(Region III—not used).} \tag{9d}$$

These balances may be simplified by neglecting the conduction fluxes ($= -k_T\,dT/dr$) at the boundaries r_i, r_c, r_d and r_0. Exact calculations by the author show that the boundary fluxes are fairly small compared to the rate of heat generated within region I where the liquid is heated to saturation, and region II where the saturated liquid evaporates. Reasonable accuracy (1.25–2.6%) was achieved by utilizing the energy balances over these two regions, combined with the overall energy balance.

Solution for the mass flux

Substituting equation (6) into (5) and integrating yields:

$$P = C_1 \ln R + C_2$$

where C_1, C_2 are two constants of integration. Utilizing (P_i, P^*) and (P^*, P_o) as the boundary-conditions for the liquid and vapor regions respectively,

yields the pressure distribution as a function of the two "interfaces" (at R_c and R_d) position:

$$P_l = 1-(1-P^*)\frac{\ln(R/R_i)}{\ln(R_c/R_i)} \tag{10a}$$

$$P_v = P^*\frac{\ln(R/R_o)}{\ln(R_d/R_o)} \tag{10b}$$

where the subscripts l and v refer to the liquid and vapor regions, respectively.

The mass flux in each region is given by:

$$\dot{m} = 2\pi r v_f \rho_f \tag{11a}$$

or in dimensionless form:

$$\dot{M} = \frac{\dot{m}}{\bar{v}\rho_f(r_o-r_i)} = 2\pi VR \tag{11b}$$

where v_f (or V) is the Darcy velocity as is defined by equation (1) [or equation (6)] and by the local pressure gradient (dp/dr). When the latter is evaluated from either of equations (10) and is combined with equations (6) and (11b) the following are obtained:

$$\dot{M}_l = \frac{\dot{m}_l}{\bar{v}_l\rho_f(r_o-r_i)} = 2\pi\frac{1-P^*}{\ln(R_c/R_i)} \tag{12a}$$

$$\dot{M}_v = \frac{\dot{m}_v}{\bar{v}_v\rho_v(r_o-r_i)} = 2\pi\frac{P^*}{\ln(R_o/R_d)}. \tag{12b}$$

Since steady-state is considered the mass fluxes in the liquid and vapor regions are identical. Equating equations (12a) and (12b) yields the saturation pressure:

$$P^* = \frac{\ln(R_o/R_d)}{\ln(R_c/R_i)+\ln(R_o/R_d)}. \tag{13}$$

Equation (13) is used now to eliminate P^* in either of equations (12). This yields the dimensionless mass flux as a function of the "interfaces" position:

$$\dot{M}_l = \dot{M}_v = 1/[\ln(R_c/R_i)+\ln(R_o/R_d)]. \tag{14}$$

Note that once the "interfaces" position R_c and R_d are known, equations (10) and (14) may be used to evaluate the pressure distributions and the mass flux through the porous medium.

Solution of the energy equation

(A) *Constant rate of heat generation.* In the case under consideration, F [in equation (7)] vanishes and equation (7) is a linear equation of the first order. Its solution is given by:

$$\theta = C_1\frac{R^D}{D} - MR^2 + C_2 \tag{15a}$$

where

$$M = \begin{cases} \frac{1}{2}\dfrac{N}{2-D} & \text{if} \quad D \neq 2 \\ \dfrac{N}{2}(\ln R - \frac{1}{2}) & \text{if} \quad D = 2. \end{cases} \tag{15b}$$

The constants of integration C_1 and C_2 are now evaluated for each of the liquid and vapor region by introducing the boundary-conditions as in equation (8). For $D \neq 2$ this yields:

$$\theta_l = [-\theta_i + M_l(R_c^2-R_i^2)]\frac{R^{D_l}-R_i^{D_l}}{R_c^{D_l}-R_i^{D_l}} - M_l(R^2-R_i^2)+\theta_i \qquad D \neq 2 \tag{16a}$$

$$\theta_v = [\theta_o + M_v(R_o^2-R_d^2)]\frac{R^{D_v}-R_d^{D_v}}{R_o^{D_v}-R_d^{D_v}} - M_v(R^2-R_d^2) \qquad D \neq 2. \tag{16b}$$

where l, v refer to the liquid and vapor region respectively, and M is as defined in (15b). The corresponding expressions for $D = 2$ are:

$$\theta_l = \left[-\theta_i + \frac{N}{2}\left(R_c^2\ln R_c - \frac{R_c^2}{2} - R_i^2\ln R_i + \frac{R_i^2}{2}\right)\right] \times \frac{R^2-R_i^2}{R_c^2-R_i^2} - \frac{N}{2}\left(R^2\ln R - \frac{R^2}{2} - R_i^2\ln R_i - \frac{R_i^2}{2}\right)+\theta_i \qquad D = 2 \tag{16c}$$

$$\theta_v = \left[\theta_o + \frac{N}{2}\left(R_o^2\ln R_o - \frac{R_o^2}{2} - R_d^2\ln R_d + \frac{R_d^2}{2}\right)\right] \times \frac{R^2-R_d^2}{R_o^2-R_d^2} - \frac{N}{2}\left(R^2\ln R - \frac{R^2}{2} - R_d^2\ln R_i - \frac{R_d^2}{2}\right)+\theta_i \qquad D = 2. \tag{16d}$$

Integration of equations (9) for constant q, normalizing and combining the first two yield:

$$R_d = [R_c^2 - \rho_{l,v}J_a(R_c^2-R_i^2)/\theta_i]^{\frac{1}{2}} \tag{17}$$

substituting $\dot{M}$ and R_d into equation (9c) yields:

$$\frac{\theta_o}{C_{p_{l,v}}} - \theta_i + J_a - \tfrac{1}{2}\frac{N}{Pev_{l,v}}\frac{R_o+R_i}{R_o-R_i} \times \{\ln[R_o/(R_c^2-\rho_{l,v}J_a(R_c^2-R_i^2)/\theta_i)] - v_{l,v}\ln(R_c/R_i)\} = 0. \tag{18}$$

Equation (18) may be solved for R_c based on known values of θ_i, θ_o and N. Starting with initial guess for R_c, equation (17) is used to evaluate R_d. Based on these R_c, R_d values, the corresponding q (of N) is calculated by equation (9b). The calculated N is inserted into equation (18), the solution of which yields a new value of r_c which is compared with the previous one, until convergence is achieved.

(B) *Temperature-dependent rate of heat generation.* In the case of temperature-dependent rate of heat generation within the porous matrix, and in view of the q-function in equation (2a), the energy equation turns to a non-linear equation in T. [$F \neq 0$ in equation (7)].

There may be a great interest in analytical solutions, both exact and approximate, which may serve to a further understanding of the physical phenomena. The author is aware of other forms for q which may lead to close-form analytical solutions. However, such solutions are limited to the specific q-function used. A general procedure for solving the present problem for a non-linear form of q [as in equation (2a)], might prove useful for a wide spectrum of q-functions. Moreover many engineering situations require fast and

accurate results. Therefore a general calculation procedure for numerically solving equation (7) is outlined in what follows:

(a) Starting the first iteration ($j = 0$) the solution of constant $q(= q_r)$ is initially assumed. Each of the regions I and III is divided into increments of $(R_c - R_i)/i$ and $(R_o - R_d)/i$, respectively. (In region II the temperature is constant.) For any iteration $j+1$, the calculated results of the previous iteration j are assumed.

(b) Utilizing the temperature profile θ_i^j from (a), average N in the first increment ($i = 0$, to $i = 1$) and use this N_{av}^j $[= (N_i^{j+1} + N_{i+1}^j)/2]$ to calculate the new temperature [equations (16)] at $i = 1$, θ^{j+1}, based on the inlet temperature at $i = 0$. The temperature at $i+1$, θ_{i+1}^{j+1} is similarly evaluated from the value at i, θ_i^{j+1}, and the average N_{av}^j over i to $i+1$.

(c) Continue b for the liquid and vapor regions, until the complete new profile θ_i^{j+1} is obtained.

(d) The integrals in equations (9) are numerically calculated and used to evaluate the new values of R_c^{j+1} and R_d^{j+1} in a similar manner as in (A).

(e) Calculate $\sum_i (\theta_i^{j+1} - \theta_i^j)^2$

(f) Repeat (b) to (e) until the value from (e) approaches a small value.

A general computer program was set for solving the temperature profiles, the "interfaces" position and the possible mass flux for a given set of operating-conditions. The value of i and j are determined along the way according to accuracy requirements and speed considerations. The program was demonstrated by utilizing the q-function in equation (2a). Some of the calculated results are presented below.

PRESENTATION OF CALCULATED RESULTS

The present study represents an attempt to introduce the inter-relations between the characteristic operation parameters of internally heated porous reactor. The parameters affecting the performance of such an element are (a) the rate of heat generation within the solid, (b) the rate of flow of working fluid, (c) the degree of pre-heating of the incoming liquid feed and (d) the degree of superheat of outcoming vapor. These are presented in the following for a constant rate of heat generation [$\beta = 0$ in equation (2a)] and for a temperature-dependent rate of heat generation. The solution of later case is iteratively obtained as detailed in previous section. Since the results are obtained numerically, it is interesting to note the point values of the basic variables R_c, R_d and the temperature profiles through the various regions.

(Note that the following calculated results were evaluated for water as a working fluid and a hollow cylindrical porous element of a constant porosity ($= 0.39$) and inner and outer radii of 1.0 cm and 2.0 cm. Thus the dimensionless radii R_i, R_o are 1.0 and 2.0 respectively.)

Figures 2–4 represent the dimensionless radial distance of the two phase-change "interfaces" for various degrees of superheat and various values of Jacob number. Note that as $\theta_o \to 0$ (or $\theta_i \to -1.0$) the outcoming vapor is at saturation temperature T^*, and hence $R_d \to R_o$, almost unaffected by the temperature-coefficient, β. However, the variation of R_c with β is by contrast more pronounced. As $\theta_o \to 1.0$ (or $\theta_i \to 0$) the incoming liquid is at the saturation temperature T^*, and hence $R_c \to R_i$, again unaffected by the temperature-coefficient, while R_d being the most affected variable.

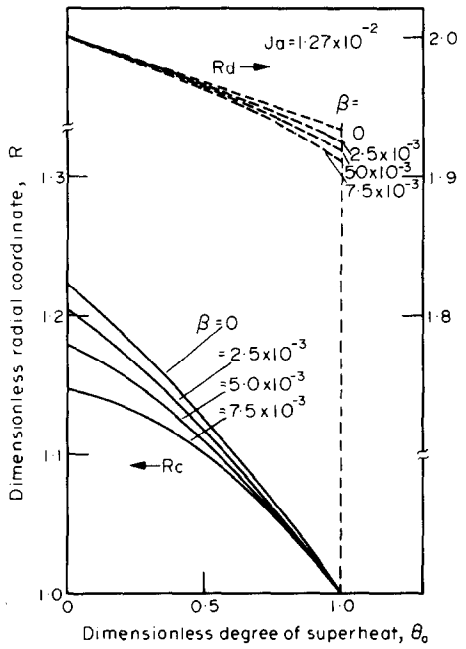

FIG. 2. Effect of temperature coefficient, β on the phase-change "interfaces" for various degrees of superheat.

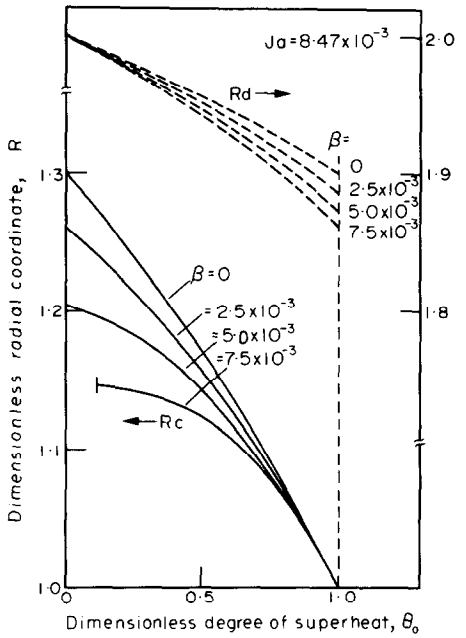

FIG. 3. Effect of temperature coefficient, β on the phase-change "interfaces" for various degrees of superheat.

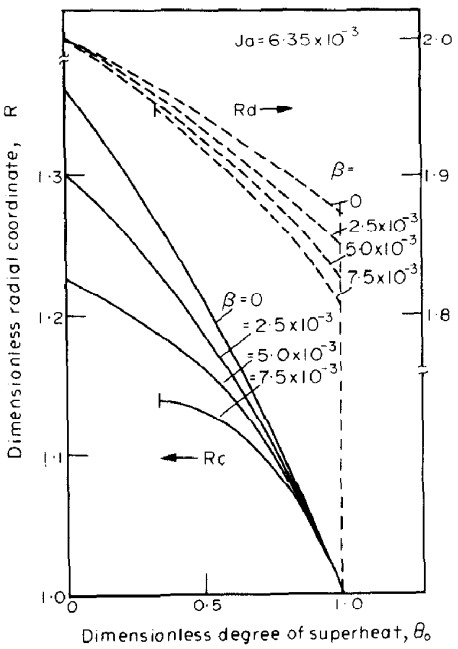

FIG. 4. Effect of temperature coefficient, β on the phase-change "interfaces" for various degrees of superheat.

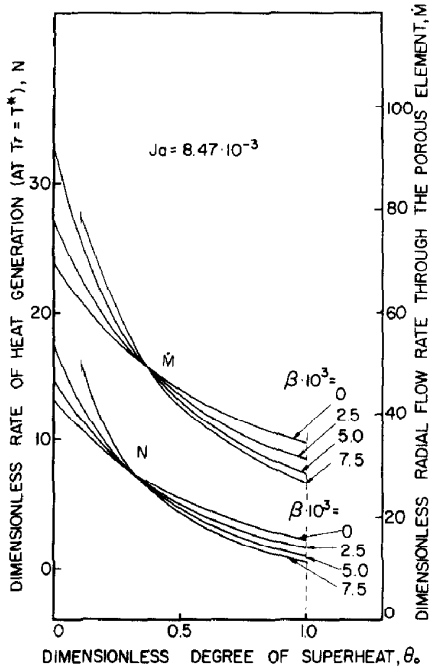

FIG. 6. Mass flow rate and heat generation rate for various β at various degrees of superheat.

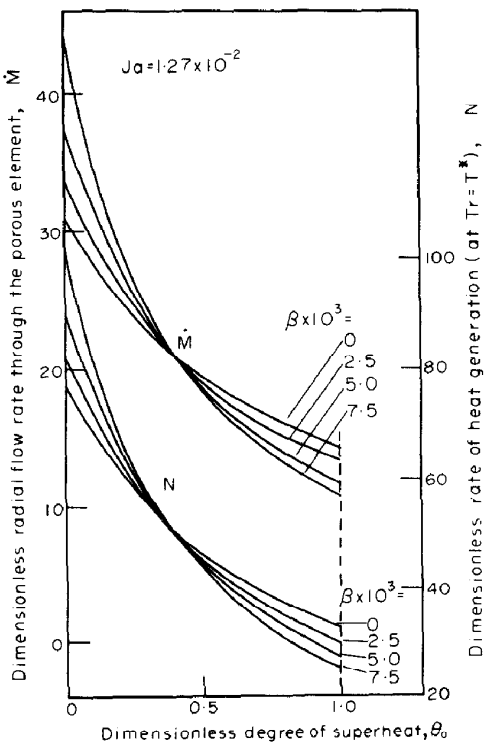

FIG. 5. Mass flow rate and heat generation rate for various β at various degrees of superheat.

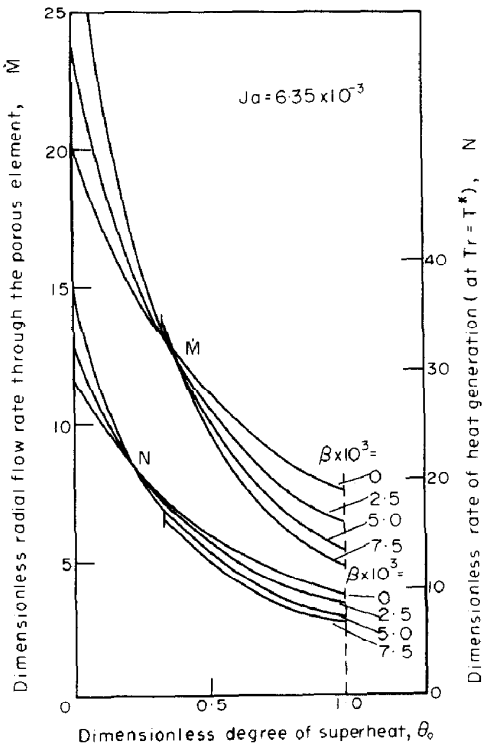

FIG. 7. Mass flow rate and heat generation rate for various β at various degrees of superheat.

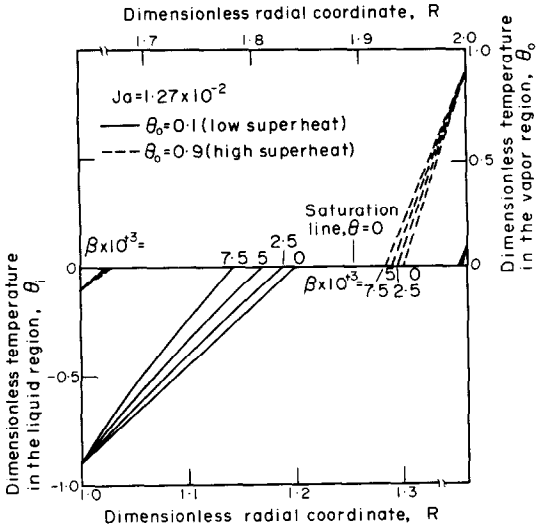

FIG. 8. Dimensionless temperature profile within the porous element.

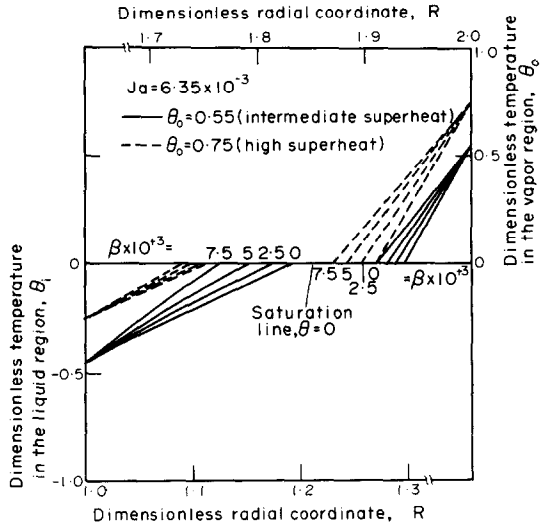

FIG. 9. Dimensionless temperature profile within the porous element.

The variation of the dimensionless mass flux with the degree of superheat and the temperature-coefficient of the heat generation is shown in Figs. 5–7. Also included in these figures the dimensionless rate of heat generation, N, corresponding to the reference temperature $T_r \equiv T^*$ [see equations (2a, 4)].

In view of equation (14), the possible mass flux through the porous element is determined by R_c and R_d. As the degree of superheat is increased, the phase-change "interface", R_d, moves inside while R_c remains almost unchanged. Thus, the mass flux and hence the required rate of heat generation should both decrease [see equation (14)]. Similarly, for low superheat, the increase in R_c brings about a corresponding increase in the mass flux.

As is demonstrated in Figs. 5–7, increasing β at low superheat yields a higher mass flux while at high superheat the mass flux is reduced by increasing β. This behavior is due to the value chosen for the reference temperature ($T_r = T^*$). (Where the reference temperature equals T_i, the increase of β should yield a decrease in the possible mass flux through the whole range.)

The mass flux and the rate of heat generation determine the values of the dimensionless groups D and M which appears in the temperature profiles. Typical temperature profiles for high, low and intermediate superheat are represented in Figs. 8–9. Note that, θ_l denotes the temperature variation in the liquid heating region and θ_v corresponds to the temperature variations in the superheating region. The intersection of θ_l and θ_v curves with the saturation line ($\theta = 0$) indicates the values of R_c and R_d, respectively.

SUMMARY AND FINAL REMARKS

A theoretical analysis of internally energised porous reactor has been presented. The concept of generating temperature-dependent rate of heat within the porous element is aimed at gaining stability in operation and protecting the solid from burnout.

In the problem discussed, liquid at a constant temperature is fed into one end of a porous medium, and a superheated (or saturated) steam is produced at the opposite end of the element. The regions of different phases are "separated" by two phase-change "interfaces". The first interface (at $R = R_c$) denotes the average radial distance where evaporation of the saturated liquid starts while the other "interface" (at $R = R_d$) denotes the average radial distance where complete evaporation is reached.

The analysis is applicable for predicting the characteristics of a porous reactor producing vapor where small quantities of variable controlled superheatings are required. The principle may also be applied for cooling of nuclear particles by evaporating high latent-heat fluid while flowing through the particles bed.

Based on various values of the temperature-coefficient, the characteristics presented so far enables one to predict the mass flux (or the reactor load) for a required degree of superheat and an available rate of heat generation.

In steady-state operation, the use of temperature-dependent rate of heat generation might cause a significant decrease in the possible mass flux through the element. This decrease is well pronounced at relatively high superheat and high Jacob number.

The analysis presented here is restricted to low flow rates of coolant through the porous medium, and is based on the temperature-dependent rate as is defined in equation (2a). Also, the upper bound of effective properties (of solid and fluid) as is given by Hashin and Shtrikteman [17] has been used. An extended analysis for high flow rate and a general function of the heat generation rate with the local temperature is now underway.

Finally one can conceive of a reactor formed by three separated cylindrical concentric elements. Gaps are incorporated between the cylindrical layers. Saturated liquid from the first element is fed to the intermediate

layer where evaporation takes place, and the saturated vapor is now fed to the superheating element. The rate of heat generation in each portion can be adjusted correspondingly to its size and the amount of energy required. Similar approach may be applied to analyse such a system.

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TRANSFERT THERMIQUE STATIONNAIRE A L'INTERIEUR D'UN MILIEU POREUX AVEC PRODUCTION DE CHALEUR DEPENDANT DE LA TEMPERATURE

Résumé—On présente l'étude théorique d'un réacteur poreux avec source interne d'énergie. Le taux de production de chaleur est supposé constant ou fonction de la température. Le fluide traversant le milieu poreux chauffé subit un changement de phase liquide–vapeur et la vapeur est ensuite surchauffée. On suppose que les régions relatives aux différentes phases sont séparées par deux interfaces de changement de phase: le premier indique la distance moyenne sur laquelle l'évaporation débute et le second indique la distance moyenne à laquelle une évaporation complète est atteinte.

On évalue les caractéristiques des divers paramètres qui agissent sur le fonctionnement d'un élément poreux creux cylindrique et les résultats sont présentés dans un domaine représentant le régime d'écoulement visqueux dans lequel les forces d'inertie sont négligées.

L'idée d'un apport de chaleur, fonction de la température à l'intérieur de l'élément poreux, vise à améliorer la stabilité de l'opération et à protéger le solide de l'assèchement.

STATIONÄRER WÄRMETRANSPORT IN PORÖSEN MEDIEN MIT TEMPERATURABHÄNGIGER WÄRMEERZEUGUNG

Zusammenfassung—Es wird eine theoretische Analyse für einen porösen Reaktor mit innerer Eigenwärmeerzeugung angegeben. Konstante und temperaturabhängige Wärmeerzeugung sind angenommen. Flüssigkeit dringt durch das erhitzte poröse Element, unterliegt einer Phasenänderung zu Dampf, und der Dampf wird weiter überhitzt. Es wird angenommen, daß die Bereiche unterschiedlicher Phasen durch zwei Phasentrennflächen abgeteilt sind: die erste gibt an, wo die Verdampfung beginnt und die zweite, wo die Verdampfung beendet ist.

Die Charakteristika für verschiedene Parameter, die die Wirkungsweise eines porösen Elements in Hohlzylinderform beeinflussen, werden ermittelt und für einen repräsentativen Bereich der Strömung zäher Flüssigkeit angegeben, wobei Trägheitskräfte vernachlässigt werden.

Die Voraussetzung temperaturabhängiger Wärmeerzeugung im porösen Element sollte der Stabilität der Arbeitsbedingungen dienen und das Festkörperteilchen vom Ausbrennen schützen.

СТАЦИОНАРНЫЙ ТЕПЛООБМЕН В ПОРИСТОЙ СРЕДЕ С ЗАВИСЯЩИМ ОТ ТЕМПЕРАТУРЫ ТЕПЛОВЫДЕЛЕНИЕМ

Аннотация — Дается теоретический анализ пористого реактора с внутренним выделением энергии. Предполагается, что скорость выделения тепла зависит от температуры. Жидкость, прошедшая через нагретый пористый элемент, превращается в пар, а далее происходит перегрев пара. Предполагается, что области различных фаз «разделяются» двумя границами, первая из которых представляет среднее расстояние, на котором начинается испарение, а вторая — среднее расстояние, на котором происходит полное испарение.

Рассчитывались различные параметры, влияющие на характеристики полого цилиндрического пористого элемента. Они представлены для диапазона вязкого режима течения с пренебрежимо малыми силами инерции.

Условие зависимости скорости выделения тепла от температуры в пористом элементе используется для достижения устойчивости режима работы и защиты твердого тела от перегорания.